Introduction

July 16, 2013

Welcome

Acknowledgements:

- Benjamin Pierce, André Scedrov, U Penn support team
- Office of Naval Research
- EasyCrypt users

Organization:

- Lectures: overview of key components
- Labs: hands-on experience
- Workshop (Friday)

School web page:

http://www.easycrypt.info/school.html

EasyCrypt in a nusthell

- EasyCrypt is a tool-assisted platform for proving security of cryptographic constructions in the computational model
- Views cryptographic proofs as relational verification of open parametric probabilistic programs

- Leverage PL and PV techniques for cryptographic proofs
- Be accessible to cryptographers (choice of PL)
- Support high-level reasoning principles (still ongoing)
- Provide reasonable level of automation
- Reuse off-the-shelf verification tools (we use Why3)

EasyCrypt usage

- EasyCrypt is generic: no restriction on
  - ☞ primitives and protocols
  - ☞ security notions and assumptions
- Can be used interactively or as a certifying back-end
  - ☞ for cryptographic compilers (ZK)
  - ☞ for domain-specific (computational or symbolic) logics
- Can verify implementations
  - ☞ C-mode
  - ☞ CompCert as a certifying back-end

Evolution

Started in 2009. One older brother (CertiCrypt), started 2006.

- At first, mostly automated proofs
- v0.2 Interactive proofs in pRHL
- v1.0 Modular proofs, all layers explicit and with support for interactive proofs

Warning

v1.0 not yet finalized. Still needs to work on

- increasing automation
- high-level proof steps
- small(er) TCB
- ...

EasyCrypt: Languages

Typed imperative language

$\mathcal{C}$	$::=$	skip	skip
	$\mid$	$\mathcal{V} = \mathcal{E}$	assignment
	$\mid$	$\mathcal{V} = \$\mathcal{D}$	random sampling
	$\mid$	$\mathcal{C};\ \mathcal{C}$	sequence
	$\mid$	if $\mathcal{E}$ then $\mathcal{C}$ else $\mathcal{C}$	conditional
	$\mid$	while $\mathcal{E}$ do $\mathcal{C}$	while loop
	$\mid$	$\mathcal{V} = \mathcal{F}(\mathcal{E}, \ldots, \mathcal{E})$	procedure call

Expression language:

- features first-class distributions α *distr*
- allows higher-order expressions
- is extensible

Semantics of programs

Discrete sub-distribution transformers

$$[\![c]\!] : \mathcal{M} \to \mathcal{M} \text{ distr}$$

Probability of an event

$$\Pr[c, m : E] = [\![c]\!]_m \, E$$

Losslessness

$$\Pr[c, m : \top] = 1$$

EasyCrypt: Logics

- Ambient higher-order logic
- Hoare Logic $c : P \Longrightarrow Q$
- Probabilistic Hoare Logic (behind compute in v0.2)

$$[c : P \Longrightarrow Q] \leq \delta \quad [c : P \Longrightarrow Q] = \delta \quad [c : P \Longrightarrow Q] \geq \delta$$

- Probabilistic Relational Hoare Logic $c_1 \sim c_2 : P \Longrightarrow Q$

☞ Logics serve complementary purposes
☞ Some overlaps, many interplays
☞ HL, pHL, pRHL embedded in ambient logic

PRHL: intuition and preview

Judgment $c_1 \sim c_2 : P \Longrightarrow Q$ is valid iff for all memories m_1 and m_2

$$P\ m_1\ m_2 \Rightarrow Q^{\#}\ [\![c_1]\!]_{m_1}\ [\![c_2]\!]_{m_2}$$

Valid judgments allow deriving probability claims; eg if $P\ m_1\ m_2$ and $c_1 \sim c_2 : P \Longrightarrow Q$ and $Q \Rightarrow A\langle 1\rangle \Leftrightarrow B\langle 2\rangle$ then

$$\Pr[c_1, m_1 : A] = \Pr[c_2, m_2 : B]$$

Example rule:

$$\frac{c_1 \sim c : P \wedge e\langle 1\rangle \Longrightarrow Q \qquad c_2 \sim c : P \wedge \neg e\langle 1\rangle \Longrightarrow Q}{\text{if } e \text{ then } c_1 \text{ else } c_2 \sim c : P \Longrightarrow Q}$$

$$\frac{\begin{array}{c} P \Rightarrow e\langle 1\rangle = e'\langle 2\rangle \\ c_1 \sim c_1' : P \wedge e\langle 1\rangle \Longrightarrow Q \quad c_2 \sim c_2' : P \wedge \neg e\langle 1\rangle \Longrightarrow Q \end{array}}{\text{if } e \text{ then } c_1 \text{ else } c_2 \sim \text{if } e' \text{ then } c_1' \text{ else } c_2' : P \Longrightarrow Q}$$

EasyCrypt: modules and theories

Modules (beware memory model)

- Instantiating generic transformations (simplified syntax)

```
forall &m (A <: AdvCCA), exists (B <: AdvCPA),
      Pr[CCA(FO(S),A) @ &m : b' = b ] <=
      Pr[CPA(S,B) @ &m : b' = b] + ....
```

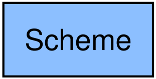
- Supporting high-level reasoning steps

Theories

- Supports code reuse
- “Polymorphism” via abstract types
- “Quantification” via abstract operators

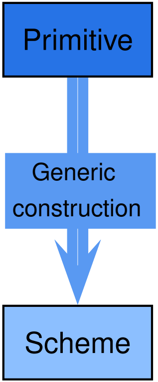
Plans to implement datatypes and type classes

Provable security



Provable security

Provable security



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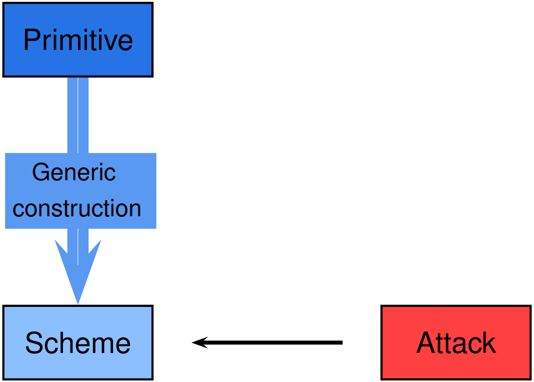

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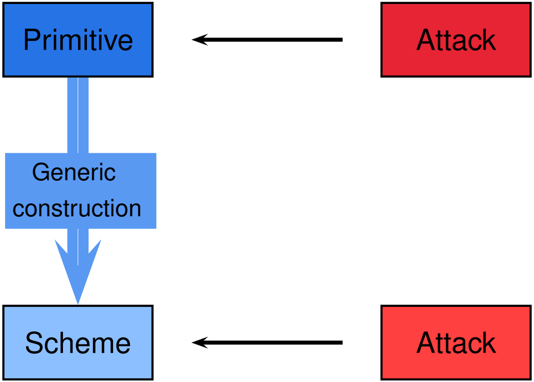

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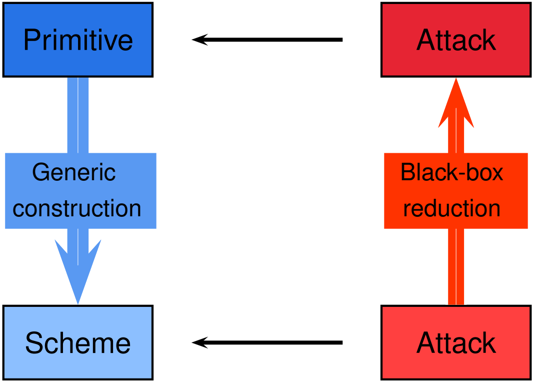

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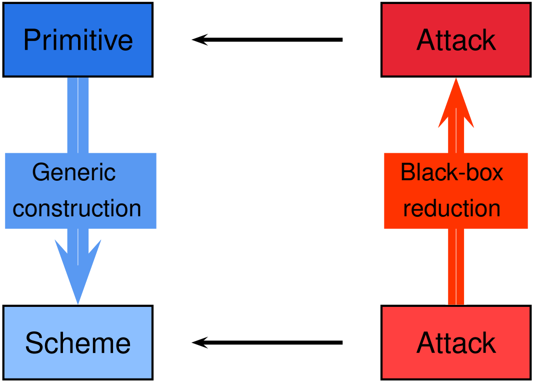


Ideally attacks have similar execution times

Public-key encryption

Algorithms $(\mathcal{K}, \mathcal{E}, \mathcal{D})$, s.t.:

- $\mathcal{E}$ takes as inputs a public key and a message, and outputs a ciphertext
- $\mathcal{D}$ takes as inputs a secret key and a ciphertext, and outputs a plaintext; $\mathcal{D}$ may be partial
- if (sk, pk) is a valid key pair, $\mathcal{D}_{sk}(\mathcal{E}_{pk}(m)) = m$

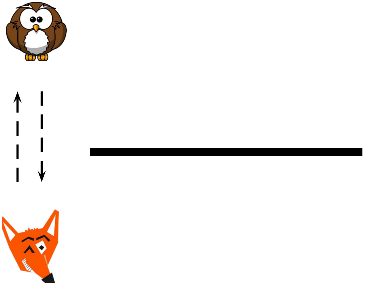
```
module type Scheme = {
  fun kg() : pkey * skey
  fun enc(pk:pkey, m:plaintext) : ciphertext
  fun dec(sk:skey, c:ciphertext) : plaintext option
}.
```

Correctness

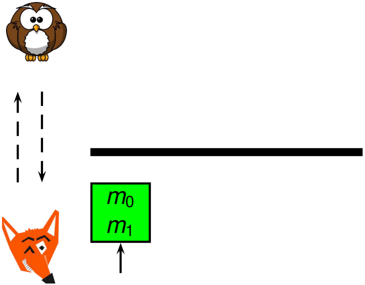
```
module Correct (S:Scheme) = {
  fun main(m:plaintext) : bool = {
    var pk : pkey;
    var sk : skey;
    var c : ciphertext;
    var m' : plaintext option;

    (pk, sk) = S.kg();
    c = S.enc(pk, m);
    m' = S.dec(sk, c);
    return (m' = Some m);
  }
}.
```

$$[Correctness(S, I) : \top \Longrightarrow \text{m'=Some m}] = 1$$

Indistinguishability



Indistinguishability



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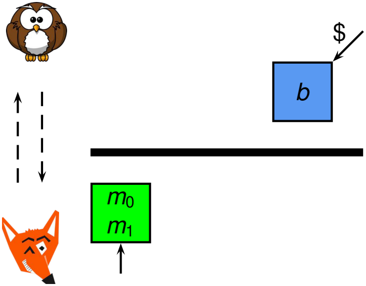

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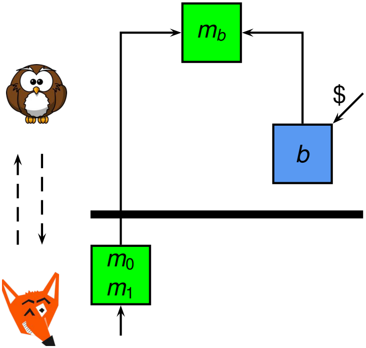

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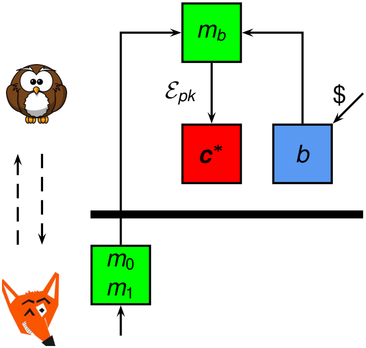

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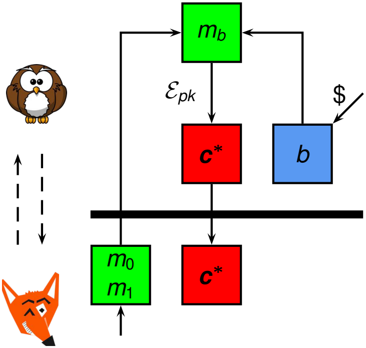

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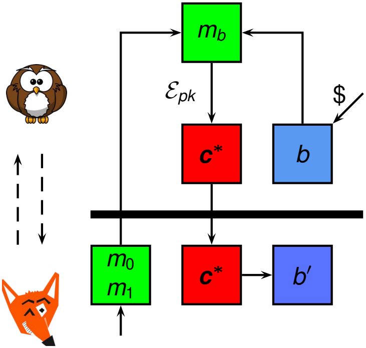

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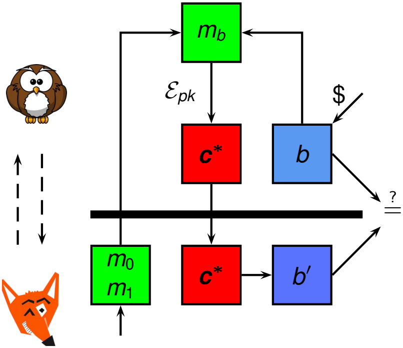

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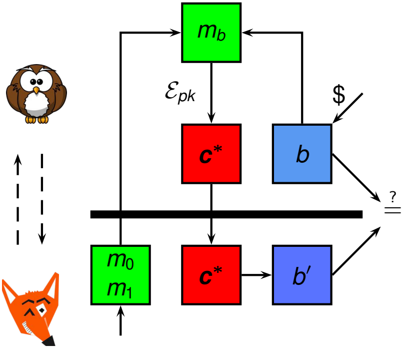


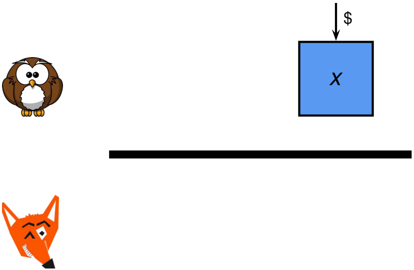
$$\left| \Pr\left[\text{IND-CCA}(\mathcal{A}) : b' = b \right] - \frac{1}{2} \right| \quad \text{small}$$

Indistinguishability

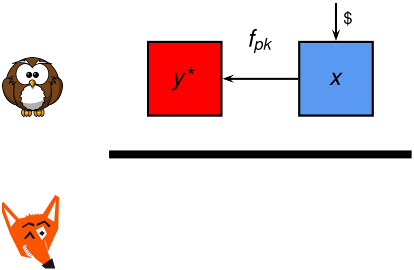
```
module CPA (S:Scheme, A:Adversary) = {
  fun main() : bool = {
    var pk : pkey;
    var sk : skey;
    var m0, m1 : plaintext;
    var c : ciphertext;
    var b, b' : bool;

    (pk, sk) = S.kg();
    (m0, m1) = A.choose(pk);
    b = ${0,1};
    c = S.enc(pk, b ? m1 : m0);
    b' = A.guess(c);
    return (b' = b);
  }
}.
```

One-way trapdoor permutations

One-way trapdoor permutations



One-way trapdoor permutations



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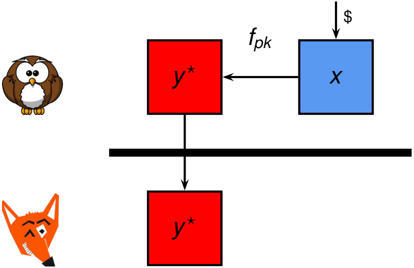

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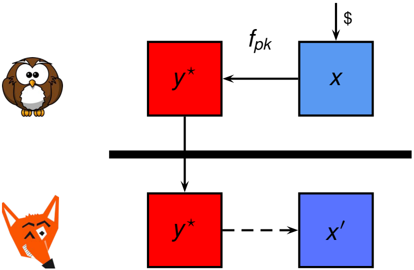

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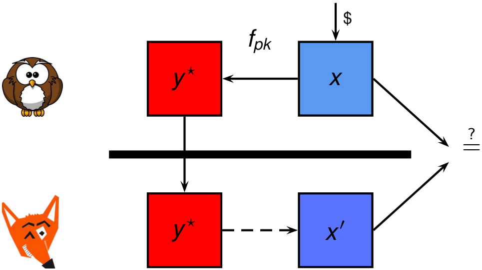

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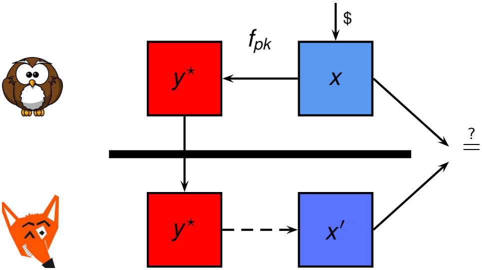

$$\Pr\left[\mathsf{OW}(\mathcal{I}) : x' = x\right] \quad \text{small}$$

One-way trapdoor permutations

```
module type Inverter = {
 fun i(pk : pkey, y : randomness) : randomness
}.

module OW(I :Inverter) ={
 fun main() : bool ={
 var x : randomness;
 var x' : randomness;
 var pk : pkey;
 var sk : skey;
   x = $uniform_rand;
   (pk,sk) = $keypairs;
   x' = I.i(pk,(f pk x));
   return (x' = x);
 }
}.
```

Random oracles (excerpts, and a bit of cheating)

```
module type Oracle =
{ fun init():unit
  fun o(x:from):to
}.

module type O_ext = { fun o(x:from):to }.

theory ROM.
  module RO:Oracle = {
    var m : (from, to) map

    fun o(x:from) : to = {
      var y : to;
      y = $dsample;
      if (!in_dom x m) m.[x] = y;
      return (m.[x]);
    }
  }.
```

Example: Bellare and Rogaway 1993 encryption

- plaintext is the type $\{0,1\}^n$ of bitstrings of length n
- randomness is the type $\{0,1\}^k$ of bitstrings of length k
- ciphertext is the type $\{0,1\}^{n+k}$ of bitstrings of length $n+k$

```
fun enc(pk:pkey, m:plaintext): ciphertext = {
  var h, s : plaintext;
  var r : randomness;

  r = ${0,1}^k;
  h = H.o(r);
  s = m ⊕ h;
  return ((f pk r) || s);
}
```

Security

For every IND-CPA adversary $\mathcal{A}$, there exists an inverter $\mathcal{I}$ st

$$\left| \Pr\left[\text{IND-CPA}(\mathcal{A}) : b' = b\right] - \frac{1}{2} \right| \leq \Pr\left[\text{OW}(\mathcal{I}) : x' = x\right]$$

Formal statement (omitting side conditions, simplified syntax)

```
forall &m (A <: Adv), exists (I <: Inverter),
    |Pr[CPA(BR,A).main() @ &m : b' = b ] − (1%r / 2%r)| <=
    Pr[OW(I).main() @ &m : x' = x].
```

Proof

Game hopping technique

Game INDCPA :
$(sk, pk) = \mathcal{K}();$
$(m_0, m_1) = \mathcal{A}_1(pk);$
$b = \$\{0,1\};$
$\boldsymbol{c^*} = \mathcal{E}_{pk}(m_b);$
$b' = \mathcal{A}_2(\boldsymbol{c^*});$
return $(b' = b);$
Encryption $\mathcal{E}_{pk}(m)$:
$r = \$\{0,1\}^\ell;$
$h = H(r);$
$s = h \oplus m;$
$c = f_{pk}(r) \parallel s;$
return $c;$

Game G :
$(sk, pk) = \mathcal{K}();$
$(m_0, m_1) = \mathcal{A}_1(pk);$
$b = \$\{0,1\};$
$\boldsymbol{c^*} = \mathcal{E}_{pk}(m_b);$
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$h = \$\{0,1\}^k;$
$s = h \oplus m;$
$c = f_{pk}(r) \parallel s;$
return $c;$

Game G$'$:
$(sk, pk) = \mathcal{K}();$
$(m_0, m_1) = \mathcal{A}_1(pk);$
$b = \$\{0,1\};$
$\boldsymbol{c^*} = \mathcal{E}_{pk}(m_b);$
$b' = \mathcal{A}_2(\boldsymbol{c^*});$
return $(b' = b);$
Encryption $\mathcal{E}_{pk}(m)$:
$r = \$\{0,1\}^\ell;$
$s = \$\{0,1\}^k;$
$h = s \oplus m;$
$c = f_{pk}(r) \parallel s;$
return $c;$

Game OW :
$(sk, pk) = \mathcal{K}();$
$y = \$\{0,1\}^\ell;$
$y' = \mathcal{I}(f_{pk}(y));$
return $(y' = y);$
Adversary $\mathcal{I}(x)$:
$(m_0, m_1) = \mathcal{A}_1(pk);$
$s = \$\{0,1\}^k;$
$\boldsymbol{c^*} = x \parallel s;$
$b' = \mathcal{A}_2(\boldsymbol{c^*});$
$y' = [z \in \boldsymbol{L}_H^{\mathcal{A}} \mid f_{pk}(z) = x];$
return y'

1. For each hop
   - prove validity of pRHL judgment
   - derive probability claim(s)
2. Obtain security bound by combining claims
3. Check execution time of constructed adversary

Conditional equivalence

$$\begin{array}{l} \mathcal{E}_{pk}(m): \\ \quad r = \$\{0,1\}^{\ell}; \\ \quad h = H(r); \\ \quad s = h \oplus m; \\ \quad c = f_{pk}(r) \parallel s; \\ \quad \text{return } c; \end{array}
\quad \blacktriangleright \quad
\begin{array}{l} \mathcal{E}_{pk}(m): \\ \quad r = \$\{0,1\}^{\ell}; \\ \quad h = \$\{0,1\}^{k}; \\ \quad s = h \oplus m; \\ \quad c = f_{pk}(r) \parallel s; \\ \quad \text{return } c; \end{array}$$

$$\text{IND-CPA} \sim \mathbf{G} : \top \Longrightarrow (\neg r \in \boldsymbol{L}_H^{\mathcal{A}})\langle 2 \rangle \quad \Rightarrow \equiv$$

$$\left|\Pr\left[\text{IND-CPA} : b' = b\right] - \Pr\left[\mathbf{G} : b' = b\right]\right| \leq \Pr\left[\mathbf{G} : r \in \boldsymbol{L}_H^{\mathcal{A}}\right]$$

Equivalence

$\mathcal{E}_{pk}(m)$:
$r = \$\{0,1\}^{\ell}$;
$h = \$\{0,1\}^{k}$;
$s = h \oplus m$;
$c = f_{pk}(r) \parallel s$;
return c;

▶

$\mathcal{E}_{pk}(m)$:
$r = \$\{0,1\}^{\ell}$;
$s = \$\{0,1\}^{k}$;
$h = s \oplus m$;
$c = f_{pk}(r) \parallel s$;
return c;

$$\mathbf{G} \sim \mathbf{G}' : \top \Longrightarrow \equiv$$

$$\Pr\left[\mathbf{G} : r \in \boldsymbol{L}_H^{\mathcal{A}}\right] = \Pr\left[\mathbf{G}' : r \in \boldsymbol{L}_H^{\mathcal{A}}\right]$$
$$\Pr\left[\mathbf{G} : b' = b\right] = \Pr\left[\mathbf{G}' : b' = b\right] = \frac{1}{2}$$

Equivalence

$\mathcal{E}_{pk}(m)$:
$r = \$\{0,1\}^{\ell}$;
$h = \$\{0,1\}^{k}$;
$s = h \oplus m$;
$c = f_{pk}(r) \parallel s$;
return c;

►

$\mathcal{E}_{pk}(m)$:
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$$\mathbf{G} \sim \mathbf{G}' : \top \Longrightarrow \equiv$$

$$\left|\Pr\left[\text{IND-CPA} : b' = b\right] - \tfrac{1}{2}\right| \leq \Pr\left[\mathbf{G}' : r \in \boldsymbol{L}_H^{\mathcal{A}}\right]$$

Reduction

Game INDCPA :
$(sk, pk) = \mathcal{K}();$
$(m_0, m_1) = \mathcal{A}_1(pk);$
$b = \$\{0,1\};$
$\boldsymbol{c}^* = \mathcal{E}_{pk}(m_b);$
$b' = \mathcal{A}_2(\boldsymbol{c}^*);$
return $(b' = b)$
Encryption $\mathcal{E}_{pk}(m)$:
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$s = \$\{0,1\}^k;$
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return c;

Game OW :
$(sk, pk) = \mathcal{K}();$
$y = \$\{0,1\}^\ell;$
$y' = \mathcal{I}(f_{pk}(y));$
return $(y' = y);$
Adversary $\mathcal{I}(x)$:
$(m_0, m_1) = \mathcal{A}_1(pk);$
$b = \$\{0,1\};$
$s = \$\{0,1\}^k;$
$\boldsymbol{c}^* = x \parallel s;$
$b' = \mathcal{A}_2(\boldsymbol{c}^*);$
$y' = [z \in \boldsymbol{L}_H^{\mathcal{A}} \mid f_{pk}(z) = x];$
return y';

$$\mathbf{G}' \sim \mathrm{OW} : \top \Longrightarrow (r \in \boldsymbol{L}_H^{\mathcal{A}})\langle 1 \rangle \Rightarrow (y' = y)\langle 2 \rangle$$

$$\Pr\left[\mathbf{G}' : r \in \boldsymbol{L}_H^{\mathcal{A}}\right] \leq \Pr\left[\mathrm{OW}(\mathcal{I}) : y' = y\right]$$

Reduction

Game INDCPA :
$(sk, pk) = \mathcal{K}();$
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return y';

$$\mathbf{G}' \sim \mathrm{OW} : \top \Longrightarrow (r \in \boldsymbol{L}_H^{\mathcal{A}})\langle 1 \rangle \Rightarrow (y' = y)\langle 2 \rangle$$

$$\left| \Pr\left[\text{IND-CPA}(\mathcal{A}) : b' = b\right] - \tfrac{1}{2} \right| \leq \Pr\left[\mathrm{OW}(\mathcal{I}) : y' = y\right]$$

Remarks

- In EasyCrypt v0.2, reasoning principles are "embedded " in pRHL proofs for the concrete construction
- In EasyCrypt v1, one can
  - ☞ prove high-level principles in an abstract setting
  - ☞ instantiate principles

  Benefits: much easier! Also favours
  - ☞ libraries of verified high-level principles
  - ☞ better proofs (shorter, faster, more robust)

Variations on IND-CPA

For every adversary $\mathcal{A}$, there exists an adversary $\mathcal{B}$ st

$$\left|\Pr\left[\text{IND-CPA}(\mathcal{A}) : b' = b\right] - \frac{1}{2}\right| = \Pr\left[\text{IND-CPA}(\mathcal{B}) : b' = b\right] - \frac{1}{2}$$

By case analysis on $\Pr[\text{IND-CPA}(\mathcal{A}) : b' = b] \leq \frac{1}{2}$

- If true, then $\mathcal{B}$ returns the result of $\mathcal{A}$
- If false, then $\mathcal{B}$ returns the negation of the result of $\mathcal{A}$

Summary

Provable security as deductive relational verification
of (open and parametrized) probabilistic programs

- EasyCrypt v1.0 is more explicit than its predecessor
- EasyCrypt v1.0 supports modular reasoning
- Shift of perspective (more instantiation, less pRHL)
- Should make tool more accessible to cryptographers